Detecting unambiguously non-Abelian geometric phases with trapped ions

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We propose for the first time an experimentally feasible scheme to disclose the noncommutative effects induced by a light-induced non-Abelian gauge structure with trapped ions. Under an appropriate configuration, a true non-Abelian gauge potential naturally arises in connection with the geometric phase associated with two degenerated dark states in a four-state atomic system interacting with three pulsed laser fields. We show that the population in atomic state at the end of a composed path formed by two closed loops C_1 and C_2 in the parameter space can be significantly different from the composed counter-ordered path. This population difference is directly induced by the noncommutative feature of non-Abelian geometric phases and can be detected unambiguously with current technology.

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Gauge structures have been used to describe almost all the fundamental interactions in nature and have been shown to play a key role in modern physics. Gauge fields are usually classified as Abelian and non-Abelian ones according to the commutative rule of the associated group elements. Remarkably, they also arise naturally in the adiabatic evolution of simple quantum systems, whose initial formulation has no apparent relationship to gauge fields[1, 2]. Berry demonstrated that the adiabatic cyclic evolution of a non-degenerated state leads to a scalar geometric phase factors corresponding to a $U(1)$ Abelian gauge field [1]. Wilczek and Zee further generalized this idea to the case of degenerate states and showed that a non-Abelian structure emerges if a set of quantum states remains degenerate as the Hamiltonian varies. When a system with N-fold degenerate levels undergos cyclic process, the mapping corresponds to a geometric phase factor with $SU(N)\times U(1)$ matrix form[2, 3]. So far, gauge structures in such simple quantum systems have found many applications in diverse fields in physics[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

Following the progress achieved in theory and application, direct experimental observations of gauge structures in quantum systems attracts increasing interests. Although Abelian geometric phases have been tested experimentally by various means [4, 5], the noncommutative effect, a key property of non-Abelian structures has not been experimentally demonstrated. In an interesting experiment[16], Tycko demonstrated the effect of geometric phase on the magnetic resonance spectrum of two-fold degenerated states[16]. However, as pointed out in Ref. [3], because of the experimental limitation in nuclear magnetic resonance (NMR), only the (commutative) Abelian part of the non-Abelian gauge structure has been experimentally observed and the direct observation of noncommutativity of non-Abelian gauge structures in NMR system is really difficult. Recent experiments in trapped-ion systems provide high-precise manipulations and measurements on internal states of atoms [17, 18, 19, 20, 21, 22, 23]. Comparing with NMR systems, true non-Abelian structures can be easier to be realized by modulating controlling parameters in principle. Thus it is significant and important to explore whether a trapped-ion system is promising for detecting the fundamental non-Abelian characteristic.

In this paper, we analyze in detail why non-Abelian structures have not been observed in NMR experiments, and propose an experimentally feasible scheme to directly detect the observable effect induced by the noncommutative property of the non-Abelian gauge potentials in a trapped-ion system[24]. We show that a true non-Abelian gauge potential naturally arises in connection with the geometric phase associated with two degenerated dark states in a four-state atomic system interacting with three pulsed laser fields. The system under consideration is a four-level atom interacting with resonant external laser pulses. Under an appropriate configuration, there exist two-fold degenerated dark states and then a non-Abelian gauge potential naturally arises. We design two specific closed loops C_1 and C_2 in the parameter space, which generate two different geometric phase factors U_1 and U_2, respectively. Consider the composite path in which one traverses first C_1 and then C_2. After a cycle, a phase factor of $U = U_2U_1$ is generated. In contrast, the other phase factor of $U' = U_1U_2$ is accumulated if one traverses first C_2 and then C_1. We find that the population of the atoms in a specific state at the end of the two evolutions U and U' can be significantly different. For instance, the population difference can be as high as 43% for the typical experimental parameters.

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Note that the observe precision of an atomic state can be higher than 99%, and thus the proposed noncommutative effect can be detected unambiguously. Since the variations of the parameters associated with the non-Abelian geometric phases can also be easily manipulated via the laser beams, the proposed scheme is very promising for detecting the fundamental non-Abelian gauge structures.

We start by reviewing a general framework of non-Abelian adiabatic geometric phase [2, 3] and its possible detection in nuclear quadrupole resonance[16]. Consider a family of Hamiltonians $H(\chi_\mu)$ ($\mu = 1, 2, \cdots, n$) depending continuously on parameters χ_μ, all of which have a set of N degenerate levels. Let η_a be an N-fold degenerate set of orthonormal instantaneous eigenstates of the Hamiltonian H described by $H|\eta_a\rangle = E|\eta_a\rangle$ with E being the eigenvalue. Under the adiabatic condition the instantaneous state $|\psi_a\rangle$ will not overflow the state vector space spanned by η_a. So $|\psi_a\rangle$ can always be expanded as a superposition of $|\eta_a\rangle$: $|\psi_a\rangle = \Sigma_b |\eta_b\rangle U_{ba}$. Substituting the wave function $|\psi_a\rangle$ into the time-dependent Schrodinger equation $i\frac{d}{dt}|\psi_a\rangle = H|\psi_a\rangle$, we have

$$\dot{U}_{ba} = -\sum_c \langle \eta_b | \dot{\eta}_c \rangle U_{ca}, \tag{1}$$

where a trivial overall dynamical phase term has been dropped. A gauge structure appears when the Hamiltonian H varies as the parameters $\chi_\mu(t)$ vary slowly with time t. We can define a gauge potential

$$A_{ab\mu} = \langle \eta_a | \frac{\partial}{\partial \chi^\mu} | \eta_b \rangle, \tag{2}$$

and have $\langle \eta_a | \dot{\eta}_b \rangle = \Sigma_\mu A_{ab\mu} (d\chi^\mu/dt)$. Integrating Eq. (2), it is straightforward to obtain

$$U_{ab} = \left[\mathcal{P} exp \left(-\int A_\mu d\chi^\mu \right) \right]_{ab}, \tag{3}$$

where $\mathcal{P}$ denotes the path-ordered operator. The quantity A defined in Eq.(2) indeed plays the role of a gauge potential and the gauge group corresponds to the unitary freedom in choosing the basis states η_a [2]. An intriguing feature of the gauge potential A lies in that it depends only on the geometry of executed path in the space of degenerate levels.

In an earlier experiment, Tycko reported the effect of geometric phase on the nuclear-quadrupole-resonance spectra[16]. The effective Hamiltonian describing Tycko's experiment is given by

$$\begin{aligned} H &= (\mathbf{S} \cdot \mathbf{B})^2 \\ &= (S_x \sin\theta \cos\varphi + S_y \sin\theta \sin\varphi + S_z \cos\theta)^2 B^2, \end{aligned} \tag{4}$$

where $\mathbf{S}$ denotes the nuclear spin operator for the sample used in the experiment (a spin-$\frac{3}{2}$ Cl atom in a $NaClO_3$ crystal is considered), and $\mathbf{B}$ represents an external magnetic field. There always exist a pairwise degeneracy of states in the parameter space because the Hamiltonian is invariant under the operation $\mathbf{S} \to -\mathbf{S}$, thus non-Abelian gauge structure may appear in the system. To address clearly the gauge structure, one can parameterize the Hamiltonian (4) as follows

$$H = B^2 e^{-i\varphi S_z} e^{-i\theta S_y} S_z^2 e^{i\theta S_y} e^{i\varphi S_z}. \tag{5}$$

As a result, the instantaneous eigenstates can be directly written as

$$|\eta_a\rangle = e^{-i\varphi S_z} e^{-i\theta S_y} |a\rangle, \tag{6}$$

where $|a\rangle$ is one of the atomic state in the set $\{|\pm\frac{3}{2}\rangle, |\pm\frac{1}{2}\rangle\}$. Here the state $|m\rangle$ represents the eigenstates of the spin operator S_z defined by $S_z|m\rangle = m|m\rangle$ and the states $|\pm m\rangle$ form a doubly degenerate sector. Substituting Eq.(6) into Eq.(2), the corresponding gauge potentials are given by

$$\begin{aligned} A_{ab\varphi} &= (-i)\langle a|(cos\theta S_z - sin\theta S_x)|b\rangle, \\ A_{ab\theta} &= (-i)\langle a|S_y|b\rangle. \end{aligned} \tag{7}$$

As clearly shown in Ref.[3], an Abelian structure occurs for $|m| = \frac{3}{2}$ sector although those states are doubly degenerate, while a true non-Abelian structure may appear for the sector $|m| = \frac{1}{2}$. Explicitly, the gauge potentials for the later case is given by,

$$A = -i\left[\left(\frac{1}{2}\sigma_z \cos\theta - \frac{\alpha}{2}\sigma_x \sin\theta\right) d\varphi + \frac{\alpha}{2}\sigma_x d\theta\right], \tag{8}$$

where $\alpha = S + 1/2$ and $\sigma_{x,y,z}$ are Pauli matrixes.

To disclose unambiguously the non-Abelian characteristic of the geometric phase, we must detect the physically observable effects induced by the noncommutativity of the gauge structure. To this end, here we consider two closed loops C_1 and C_2, both starting and ending at the same point χ_0 in the parameter space, where the two corresponding evolution operators are denoted as U_1 and U_2, respectively. For the composite path that the system first traverses along loop C_1 and then C_2, the total phase factor of $U = U_2 U_1$ is generated. If the counter-ordered evolution is executed, i.e., an evolution with first loop C_2 and then C_1, the phase factor generated is then given by that of $U^{'} = U_1 U_2$. In a general experiment, if the observable effects are different for the phase factors of U and $U^{'}$, then Non-Abelian geometric structures are confirmed there. Otherwise, only the Abelian part of the gauge structure is observed even if the quantum states of the system are degenerate.

From the above discussion, it is easy to understand that only the Abelian part of the gauge structures has been experimentally observed in the Tycko's experiment, as has been addressed in Ref.[3]. In the Tycko's experiment, the angle θ is fixed as a constant ($\cos\theta = 1/\sqrt{3}$). It is due to that only the rotation around a fixed axis can be implemented fast enough to generate observable phase shift in NMR systems. In this case gauge structure A is proportional to a fixed matrix $\cos\theta\frac{\sigma_3}{2} - \alpha \sin\theta\frac{\sigma_1}{2}$ and the evolution operator can be

written as $U = \exp[-i\varphi(\sigma_z/\sqrt{3} - \sigma_x)]$. Thus the U and gauge potential A in different time sequences are always commutable. From this point of view, we say that the gauge potential loses its non-Abelian characteristic and is of Abelianization. In order to demonstrate the non-Abelian characteristic in nuclear quadrupole resonance, the system should traverse a non-Abelian path in which both θ and φ vary with time. Unfortunately, it is difficult in an NMR setup.

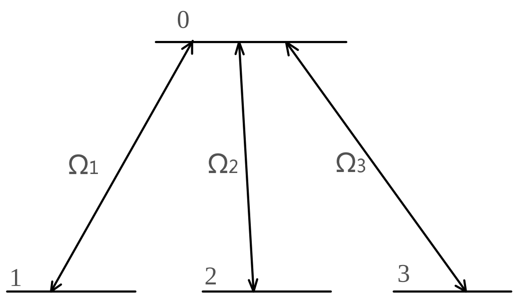

FIG. 1: Schematic map of the four-level system interacting with resonant laser fields. States $|0\rangle$ and $|i\rangle$ $(i = 1, 2, 3)$ are coupled by a resonant external laser pulse with effective Rabi frequency Ω_i.

Recently, with the rapid experimental progress in trapped ions [17, 18, 19, 20, 21, 22, 23], it is quite possible to experimentally observe the non-Abelian characteristic of gauge potentials with current technology. We now start to present a scheme to detect the non-Abelian characteristic of the gauge potentials in such system. The system under consideration is an ion (or a set of ions) confined in a (linear) Paul trap. Each ion has three ground (or metastable) states $|1\rangle$, $|2\rangle$, $|3\rangle$, and one excited state $|0\rangle$. The ground states could be different hyperfine levels or in the same manifold but with different Zeeman sublevels, and they are coupled to the excited state $|0\rangle$ by resonant classical lasers with Rabi frequencies Ω_1, Ω_2 and Ω_3, respectively, as shown in Fig.1[12]. The effective Hamiltonian for each ion in the rotating frame reads

$$H_I = -\hbar(\Omega_1|0\rangle\langle 1| + \Omega_2|0\rangle\langle 2| + \Omega_3|0\rangle\langle 3|) + H.c. \quad (9)$$

In a general case the Rabi frequencies Ω_μ can be parameterized with angle and phase variables according to $\Omega_1 = \Omega \sin\theta \cos\phi e^{i\varphi_1}$, $\Omega_2 = \Omega \sin\theta \sin\phi e^{i\varphi_2}$, and $\Omega_3 = \Omega \cos\theta e^{i\varphi_3}$, where $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}$. Two adiabatic eigenstates of H_I with zero eigenvalue (dark state) are then found to be

$$|D_1\rangle = \sin\phi e^{i\varphi}|1\rangle - \cos\phi e^{i\varphi}|2\rangle,$$

$$|D_2\rangle = \cos\theta\cos\phi e^{i\varphi}|1\rangle + \cos\theta\sin\phi e^{i\varphi}|2\rangle - \sin\theta|3\rangle, \quad (10)$$

where we have assumed that the laser phases satisfy the relations $\varphi = \varphi_3 - \varphi_1 = \varphi_3 - \varphi_2$. Substituting Eq.(10) into the formula (2), we have,

$$\begin{aligned} A_\theta &= 0, \\ A_\phi &= \begin{pmatrix} 0 & -\cos\theta \\ \cos\theta & 0 \end{pmatrix}, \\ A_\varphi &= \begin{pmatrix} i & 0 \\ 0 & i\cos^2\theta \end{pmatrix}. \end{aligned} \quad (11)$$

Then it is straightforward to find that the gauge potentials are given by

$$\begin{aligned} A &= \Sigma_\mu A_\mu d\chi^\mu \\ &= i\left(\frac{1+\cos^2\theta}{2} I + \frac{sin^2\theta}{2}\sigma_z\right) d\varphi - i\sigma_y \cos\theta d\phi, \end{aligned} \quad (12)$$

where I is a 2×2 unit matrix. Similar to the above analysis in NMR system, the non-Abelian characteristic occurs only when both the angle ϕ and phase φ vary with time. For instance, if the phase φ is fixed, we have

$$U = \exp(i\gamma\sigma_y) = \begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix}.$$

where $\gamma = \int \cos\theta d\phi$. This evolution and its geometric phase have been adressed in Ref. [26]. However, it is clear that the non-Abelian characteristic (noncommutativity) of the potential A is lost in this specific case. Therefore, rather than non-Abelian geometric phase factor, the scheme proposed in [26] can only be used to observe the Abelian part of the geometric phase, although the two-fold dark states remain degenerate as the Hamiltonian varies with time.

To observe the true non-Abelian structure, we propose two specific non-Abelian closed loops C_1 and C_2 in which both the phase φ and angle ϕ vary with time. In the closed loop C_1, we assume that the Rabi frequencies are given by

$$\begin{aligned} &\Omega_1 = \Omega_0 f(t), \\ &\Omega_2 = \Omega_0 f^2(t), \\ &\Omega_3 = \Omega_0 e^{-t^2/\tau^2} e^{i\varphi}, \end{aligned} \quad (13)$$

where $\varphi = \frac{\pi}{\tau}t$, and $f(t)$ is set as

$$f(t) = \begin{cases} \cos(\frac{\pi t}{2\tau}), & -\tau \le t \le \tau, \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

In this case, the parameters $\theta(t)$ and $\phi(t)$ are determined by

$$\begin{aligned} \tan\phi(t) &= \frac{|\Omega_2|}{|\Omega_1|} = \cos\left(\frac{\pi t}{2\tau}\right), \\ \tan\theta(t) &= \sqrt{\frac{|\Omega_1|^2 + |\Omega_2|^2}{|\Omega_3|^2}} = \sqrt{\frac{\cos^2\left(\frac{\pi t}{2\tau}\right) + \cos^4\left(\frac{\pi t}{2\tau}\right)}{\exp(-\frac{2t^2}{\tau^2})}}. \end{aligned} \quad (15)$$

From time $-\tau \to \tau$, the parameters (θ, ϕ, φ) vary from $(0, 0, -\pi)$ to $(0, 0, \pi)$, and it just accomplishes a closed

path in the parameter space. It is remarkable that all parameters (θ, ϕ, φ) are completely independent of the laser parameters Ω_0 and τ, provided that the adiabatic approximation is reasonable, thus it is easy to design the required laser pulses. By substituting Eq. (15) into Eq. (12), the gauge potential A_1 generated by the loop C_1 can be derived, and the associated evolution operator is given by

$$U_1 = \mathcal{P}exp\left(-\oint_{c_1} A_1^{\mu} d\chi^{\mu}\right). \tag{16}$$

We set the Rabi frequencies for the loop C_2 as follows,

$$\begin{aligned}\Omega_1 &= \Omega_0 f(t),\\ \Omega_2 &= \alpha\Omega_0 f^2(t),\\ \Omega_3 &= \Omega_0 e^{-(t-\beta\tau)^2/\tau^2} e^{i\varphi},\end{aligned} \tag{17}$$

where α and β are two variables to differentiate the loops C_1 and C_2, and β can also be considered as a time delay among pulses Ω_3 and Ω_1, Ω_2. In this loop both the start and end points in the parameter space (θ, ϕ, φ) are the same as those in the loop C_1. Similarly, a gauge potential A_2 will be generated in this loop C_2, and the associated evolution operator is given by

$$U_2 = \mathcal{P}exp\left(-\oint_{c_2} A_2^{\mu} d\chi^{\mu}\right). \tag{18}$$

The analytical result of integral in Eq. (16) (or Eq.(18)) is difficult in a general case, because each segment in the time-ordered exponential loop integral does not commute with the next. Nevertheless, the integral can be evaluated easily by numerical methods.

We now start to show that the composite path formed by the loops C_1 and C_2 has noncommutative feature. It is straightforward to check that the initial eigenstates $|D_{1,2}\rangle_i$ and the final eigenstates $|D_{1,2}\rangle_f$ for both pathes C_1 and C_2 are given by $|D_1\rangle_i = |D_1\rangle_f = -|2\rangle$ and $|D_2\rangle_i = |D_2\rangle_f = |1\rangle$. We may initially prepare the system as $|\Psi\rangle_i = |D_2\rangle_i$. Then we let the effective Hamiltonian undergo a composite path, first C_1 and then C_2. The total evolution operator is a two-by-two matrix denoted as $U = U_2 U_1$. After this evolution, the final state becomes

$$|\Psi\rangle_f = U_{12}|D_1\rangle_f + U_{22}|D_2\rangle_f = -U_{12}|2\rangle + U_{22}|1\rangle, \tag{19}$$

where U_{12}, U_{22} are the elements of the matrix U. On the other hand, if we let the Hamiltonian tracks the counter-ordered path, i.e., first C_2 and then C_1, the final state becomes

$$|\Psi^{'}\rangle_f = -U_{12}^{'}|2\rangle + U_{22}^{'}|1\rangle, \tag{20}$$

where $U_{12}^{'}$ and $U_{22}^{'}$ are the elements of $U^{'} = U_1 U_2$. Denote $P = |U_{22}|^2$ and $P^{'} = |U_{22}^{'}|^2$, which correspond to the possibilities to find the final state in the state $|1\rangle$ after composite paths C_2C_1 and C_1C_2, respectively, the population difference P_d between the two composite paths is given by

$$P_d = P' - P = |U_{22}^{'}|^2 - |U_{22}|^2. \tag{21}$$

The most essential feature of non-Abelian gauge structure (the noncommutativity character) can be unambiguously observed if the population difference $P_d \neq 0$.

To show clearly the observable effect induced by the noncommutativity feature of the non-Abelian geometric phases, we here numerically calculate the population difference described by Eq.(21) for some typical parameters. The populations P, P', and the difference P_d between them as functions of the parameters α and β are plotted in Fig. 2 (a) and (b). In the calculations for the evolution U (U'), we let U_1 (U_2) implement during time $-\tau \to \tau$, while U_2 (U_1) implement from τ to 3τ. The effects induced by the noncommutative property are clear since the difference P_d can be non-zero. To have a quantitative idea about the difference between the evolution U and U', the population difference P_d as a function of β for $\alpha = 7$ is shown in Fig.2(c). In this case $P_d = 43.2\%$ is derived for $\beta = 0.5$.

Nowadays the measurement technique of population of atomic states has been well developed and the detection precision can be higher than 99%. This high-precision measurement really makes it experimentally feasible to directly "see" the noncommutativity of non-Abelian gauge potentials by measuring the populations of atomic states.

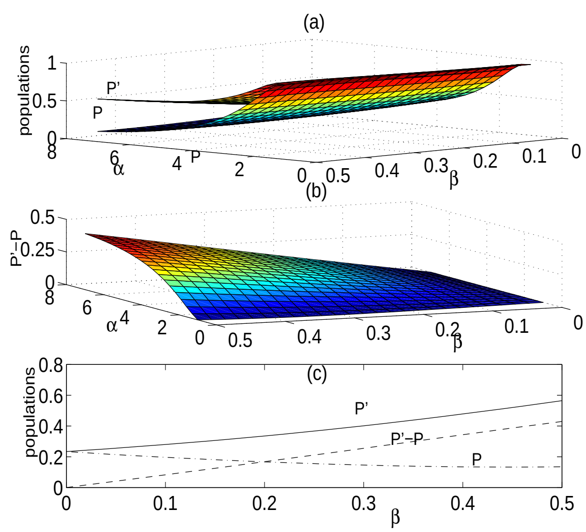

FIG. 2: (Color online) (a) The populations of atomic state $|1\rangle$ versus variables α and β. P (P') is the population of the state $|1\rangle$ at the end of the evolution U ($U^{'}$). (b) The difference $P' - P$ of the two populations shown in (a) as a functions of the parameters α and β. (c) The populations P, P' and $P' - p$ versus the parameter β when $\alpha = 7$.

We now turn to demonstrate that the adiabatic condition crucially required in the proposed scheme is well sat-

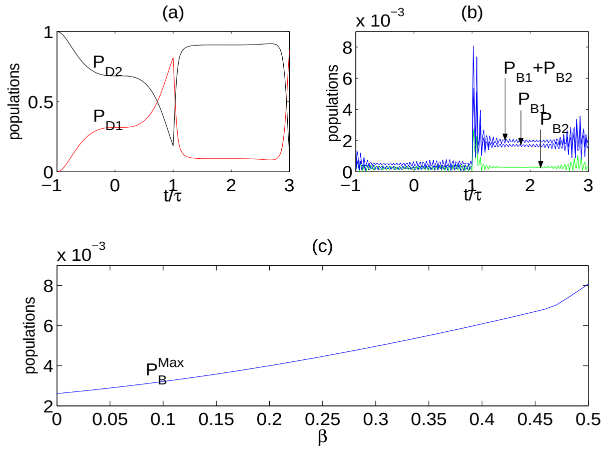

FIG. 3: (Color online) (a) The populations P_{D1} (P_{D2}) for the dark state $|D_1\rangle$ ($|D_2\rangle$) during a complete composite evolution U. (b) The populations P_{B1} (P_{B2}) for the bright state $|B_1\rangle$ ($|B_2\rangle$). (c) The maxima P_B^{Max} of the sum P_{B1} and P_{B2} during the whole evolution time versus parameters β. The parameters $\Omega_0 = 100$ and $\tau = 2$.

isfied. Besides the two degenerate dark states described in Eq.(10), there exist two other bright states with eigenvalues $\pm\Omega$ in the system described by the Hamiltonian (9), and the corresponding eigenstates are given by ,

$$\begin{aligned}|B_1\rangle = & \frac{1}{\sqrt{2}}[\sin\theta\cos\phi e^{i\varphi}|1\rangle + \sin\theta\sin\phi e^{i\varphi}|2\rangle \\ & + \cos\theta|3\rangle + e^{i\varphi}|0\rangle], \\ |B_2\rangle = & \frac{1}{\sqrt{2}}[\sin\theta\cos\phi e^{i\varphi}|1\rangle + \sin\theta\sin\phi e^{i\varphi}|2\rangle \\ & + \cos\theta|3\rangle - e^{i\varphi}|0\rangle]. \end{aligned} \quad (22)$$

The adiabatic approximation is well satisfied if the overflow from the initial dark states to the two bright states (22) can be negligible during the whole evolution. In view of this point, we numerically calculate the populations of all dark and bright states. The results for the parameters $\alpha = 7$ and $\beta = 0.5$ are plotted in Fig. 3(a) and (b). It is shown that the populations for both bright states can be negligible. In particular, we calculate the maxima (P_B^{Max}) of the sum P_{B1} and P_{B2} during the whole evolution time versus the parameter β with the other parameters being the same as those in Fig.2(c). We note that the sum of the populations for the two bright states are always below 1.0% during the whole evolution. Therefore it clearly shown that the adiabatic approximation used in the calculations of data in Fig.2 (c) is reasonable. Usually the adiabatic approximation is well-satisfied when $\Omega_0\tau \gg 1$ [26, 27].

In summary, we have proposed an experimentally feasible scheme to detect a true non-Abelian geometric phase effect in trapped ions coupled with laser beams. By designing two specific composite cyclic evolutions formed by U_2U_1 and U_1U_2, we show in detail how the effect induced by the noncommutativity of non-Abelian gauge structures can be observed through detecting the population of the internal states of trapped ions. The high-precision detection of the atomic states recently developed in the field of quantum information opens the great possibility to directly "see" the noncommutative feature of the gauge potentials. It is quite promising for experimentalists to implement the interest idea presented in this work.

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[1] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[2] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).
[3] A. Zee, Phys. Rev. A **38**, 1 (1988).
[4] A. Shapere and F. Wilczek, Geometric phases in physics (World Scientific, Singapore, 1989).
[5] P.G.Kwiat and R. Y. Chiao, Phys. Rev. Lett. **66**, 588 (2000).
[6] I. I. Satija, D. C. Dakin, and C. W. Clark, Phys. Rev. Lett. **97**, 216401 (2006).
[7] J. Ruseckas, G. Juzeliunas, P. Ohberg, and M. Fleischhauer, Phys. Rev. Lett. **95**, 010404 (2005).
[8] K. Osterloh, M. Baig, L. Santos, P. Zoller, and M. Lewenstein , Phys. Rev. Lett. **95**, 010403 (2005).
[9] S. L. Zhu, H. Fu, C.-J. Wu, S.-C. Zhang, and L.-M. Duan, Phys. Rev. Lett. **97**, 240401 (2006); Y. Li, C. Bruder, C. P. Sun, *ibid.*, **99**, 130403 (2007); S. L. Zhu, *ibid.* **96**, 077206 (2006).
[10] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999); J. Pachos, P. Zanardi, and M. Rasetti, Phys. Rev. A **61**, 010305 (2000).
[11] E. Sjoqvist, A.K. Pati, A. Ekert, J.S. Anandan, M. Ericsson, D.K.L. Oi and V. Vedral, Phys. Rev. Lett. **85**, 2845 (2000); J. Du, P.Zou, M.Shi, L.C. Kwek, J.-W.Pan, C.H. Oh, A.Ekert, D. K. L. Oi, M.Ericsson, *ibid.*, **91**, 100403 (2003); X. X. Yi, L. C. Wang, and T. Y. Zheng, *ibid.*, **92**,150406 (2004).
[12] L. M. Duan, J. I. Cirac, and P. Zoller, Science **292**, 1695 (2001).
[13] S. L. Zhu and Z. D. Wang, Phys. Rev. Lett.**91**, 187902 (2003);*ibid.*, **89**, 097902 (2002); X. D. Zhang, Q. Zhang, and Z. D. Wang, Phys. Rev. A **74**, 034302 (2006).
[14] Y. Li, P. Zhang, P. Zanardi, and C. P. Sun, Phys. Rev. A **70**, 032330 (2004).
[15] A. Recati T. Calarco, P. Zanardi, J. I. Cirac, and P. Zoller, Phys. Rev. A **66**, 032309 (2002).
[16] R. Tycko, Phys. Rev. Lett. **58**, 2281 (1987).
[17] C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **75**, 4714 (1995).

[18] D. J. Wineland, C. Monroe, W.M. Itano, D. Leibfried, B. E. King, D. M. Meekhof, J. Res. Natl. Inst. Stand. Tech. **103**, 259 (1998).

[19] P. A. Barton, C. J. S. Donald, D. M. Lucas, D. A. Stevens, A. M. Steane, and D. N. Stacey, Phys. Rev. A, **62**, 032503 (2000).

[20] D. L. Moehring, B. B. Blinov, D. W. Gidley, R. N. Kohn, Jr., M. J. Madsen, T. D. Sanderson, R. S. Vallery, and C. Monroe, Phys. Rev. A **73**, 023413 (2006); M. Acton, K.-A. Brickman, P. C. Haljana, P. J. Leeb, L. Deslauriersc, and C. Monroe, arXiv:quant-ph/0511257v2.

[21] R. Ozeri, C. Langer, J. D. Jost, B. DeMarco, A. Ben-Kish, B. R. Blakestad, J. Britton, J. Chiaverini, W. M. Itano, D. B. Hume, D. Leibfried, T. Rosenband, P. O. Schmidt, and D. J. Wineland, Phys. Rev. Lett. **95**, 030403 (2005); A. Ben-Kish, B. DeMarco, V. Meyer, M. Rowe, J. Britton, W. M. Itano, B. M. Jelenkovi, C. Langer, D. Leibfried, T. Rosenband, and D. J. Wineland, Phys. Rev. Lett. **90**, 037902 (2003).

[22] A. Kreuter, C. Becher, G. P. T. Lancaster, A. B. Mundt, C. Russo, H. Haffner, C. Roos, J. Eschner, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. **92**, 203002 (2004).

[23] P. Rabl, V. Steixner, and P. Zoller, Phys. Rev. A **72**, 043823 (2005).

[24] The scheme proposed here are equally well for cold atoms since the effective Hamiltonian for an atom interacting with three laser pulses may be exactly the same with the Hamiltonian (9).

[25] H. Goto and K. Ichimura, Phys. Rev. A **74**, 053410 (2006); H. Goto and K. Ichimura, Phys. Rev. A **75**, 033404 (2007).

[26] R. G. Unanyan, B. W. Shore, and K. Bergmann, Phys. Rev. A **59**, 2910 (1999); R. G. Unanyan and M. Fleischhauer, Phys. Rev. A **69**, 050302(R) (2004).

[27] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).